PARAMETERS OF A TURBULENT BOUNDARY LAYER AT A PERMEABLE SURFACE WITH PRONOUNCED ROUGHNESS

V. M. Epifanov and V. I. Gus'kov

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On the basis of an empirical law of the drag at a rough porous plate, the Kutateladze–Leont'ev method of calculating the turbulent dynamic boundary layer at a smooth surface is extended to the case of flow around a surface in conditions of pronounced roughness.

The method widely used at present to calculate the parameters of a turbulent dynamic boundary layer at a permeable curvilinear surface is that of Kutateladze and Leont'ev [1], which is simple and clear, and agrees satisfactorily with experiment over a wide range of experimental conditions. This method, however, was developed for the case of flow around aerodynamically smooth surfaces, which limits its usefulness in technical applications characterized by high levels of roughness in the flow surfaces.

The specific case in which pronounced roughness appears is encountered, in particular, in calculating the aerodynamic characteristics of the blade units in turbines with porous cooling. In this form of cooling, the coolant, e.g., air, passes through microchannels (pores) in the contoured shell of the blade to the outer surface of the blade and, mixing with the hot gas of the main flow, forms a protective heat-insulating layer. Cooling of the blade also occurs as a result of heat withdrawal by the coolant inside the porous walls. However, the operating efficiency of a turbine using this method of cooling may be significantly impaired as a result of the deterioration in aerodynamic characteristics of the blade units associated with the above mentioned interaction of the coolant crossflow with the main gas flow [2, 3]. As shown in theoretical work [4], the increased roughness of the porous-blade surface is another cause of increased energy loss in such blade units.

For porous materials the concept of "roughness" – like the concept of a "wall surface" ($y = 0$) – is arbitrary in nature. In [5] it was proposed that the height of elements of roughness (protuberances) be measured from an arbitrary plane for which the longitudinal component of the mean velocity is zero – $u = 0$. Sometimes, in treating experimental data, a plane passing through the protuberance vertex is taken as the origin of measurements [6]. In the present work, the surface of the wall is identified with the midline of the profilogram, i.e., is a statistical concept. The degree of roughness is estimated from the height of the irregularities measured at ten points [7]

$$R_z = \frac{\left|\sum_{i=1}^{5} H_{i_{\max}}\right| + \left|\sum_{i=1}^{5} H_{i_{\min}}\right|}{5}. \tag{1}$$

Here $H_{i\,max}$ and $H_{i\,min}$ are the absolute heights of the five tallest convexities and the absolute depths of the five largest depressions within the limits of the baseline at the measurement surface.

Depending on the type of metallic gauze used in preparing the porous material for the turbine blades, the value of R_z was found to vary in the range from 29 to 94 μm. This variation in roughness leads to a pronounced variation in the value of the contour losses [3].

Calculation of the boundary-layer parameters is hindered by the multiplicity of geometrical parameters determining the roughness (the height of the protuberances, the density of their distribution, the configuration, etc.), and also by the complexity of the effects to which they correspond. The parameter at present regarded as the most reliable for the quantitative estimation of the hydraulic effect of roughness is the "equivalent" (sand) roughness k_s, which gives both the drag and the much-studied technical roughness, determined, for example, by the value of R_z.

To determine k_s, special measurements were made of the velocity profile at plates of porous material

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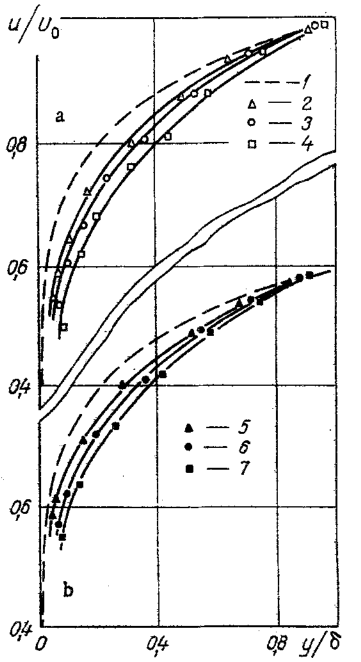

Fig. 1

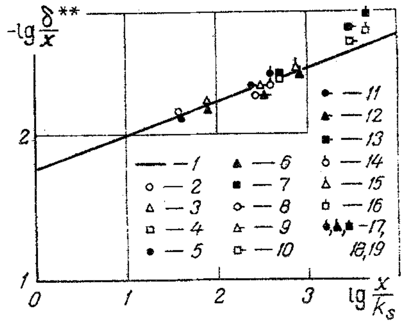

Fig. 2

Fig. 1. Experimental velocity profiles at a rough plate with R_z = 94 μm; U_0 = 36.6 m/sec (a) and 16.2 m/sec (b): 1) standard velocity profile; 2) x_{ro} = 0.02 m; 3) 0.09; 4) 0.16; 5) 0.02; 6) 0.09; 7) 0.16.

Fig. 2. Analysis of the experimental results in the form of a dependence of $\log \delta^{**}/x$ on $\log x/k_s$; 1) approximation of the experimental results; 2-4) measurements at a porous plate (R_z = 94 μm) with x_{ro} = 0.02, 0.09, and 0.16 m, respectively, and U_0 = 36.6 m/sec; 5-7) the same for U_0 = 16.2 m/sec; 8-10) measurements at a porous plate (R_z = 54 μm) with x_{ro} = 0.02, 0.09, and 0.16 m and U_0 = 36.6 m/sec; 11-13) the same for U_0 = 16.2 m/sec; 14-16) measurements at a porous plate (R_z = 29 μm) with x_{ro} = 0.02, 0.09, and 0.16 m and U_0 = 36.6 m/sec; 17-19) the same for U_0 = 16.2 m/sec.

(without blowing of coolant) with R_z = 29, 54, and 94 μm. The measurements were made with incoming-flux velocities U_0 = 16.2 and 36.6 m/sec. Experimental velocity profiles in a boundary layer at a wall surface with R_z = 94 μm obtained in three different cross sections are shown in Fig. 1. The experimental results are sufficiently well approximated by a power-law velocity profile of the type $u/U_0 = (y/\delta)^n$ (see Fig. 1). Choosing the value of n and measuring the boundary-layer thickness δ, k_s may be determined from the relation [8]

$$n = \left[1 - \frac{2.5}{8.48 - 5.75 \lg \frac{k_s}{\delta}} \right]^{-1}. \tag{2}$$

From the results of six measurements (at three cross sections for two values of the velocity U_0), the mean value k_s = (389 + 393 + 390 + 392 + 384 + 392)/6 = 390 μm for a plate with R_z = 94 μm; k_s = 240 μm for R_z = 54 μm and k_s = 40 μm for R_z = 29 μm.

By expressing the results as a dependence of δ^{**}/x on x/k_s (Fig. 2), a relation may be determined for calculating the change in momentum-loss surface δ^{**} from the plate roughness surface, in the form

$$\delta^{**} = 0.0202\, x \left(\frac{x}{k_s} \right)^{-0.228}. \tag{3}$$

It is characteristic that the deviation of the experimental values from the approximating relation in Eq. (3) is only observed for a plate with k_s = 40 μm, this discrepancy increasing over the length of the plate and with decrease in the velocity U_0. This is evidently explained by the decrease in relative roughness k_s/δ as a result of increase in δ over the plate roughness length. The effect of the velocity U_0 (for Re_0) on δ^{**} indicates that,

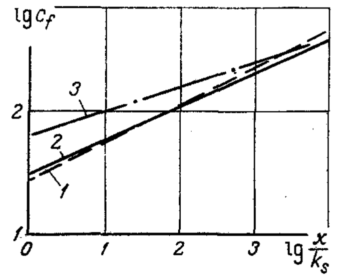

Fig. 3

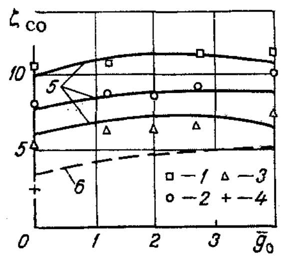

Fig. 4

Fig. 3. Comparison of values of the friction coefficient calculated: 1) from the relation $c_f = 0.0318\ (x/k_s)^{-0.228}$; 2) from $c_f = 0.0363\ (k_s/x)^{0.251}$ [7]; 3) from $c_f = 0.0139\ (x/k_s)^{-1/7}$ [8].

Fig. 4. Comparison of calculated and experimental values of the contour-loss coefficient in a turbine lattice with coolant injection through the permeable contour surface: 1-3) experimental values for porous contours with a degree of roughness R_z = 94, 54, and 29 μm, respectively [3]; 4) aerodynamically smooth impermeable profile [3]; 5) results of calculations taking into account the roughness of the porous material; 6) calculation by the method of [1].

beginning with the second cross section (x ≈ 90 mm), the flow around the surface does not correspond in type to conditions with the total appearance of roughness.

Differentiating Eq. (3) with respect to the longitudinal coordinate x gives

$$\frac{d\delta^{**}}{dx} = 0.0159 \left(\frac{x}{k_s}\right)^{-0.228} \tag{4}$$

or in accordance with the integral relation for flow at a plate

$$c_f = 0.0318 \left(\frac{x}{k_s}\right)^{-0.228}. \tag{5}$$

Results for the local friction coefficient c_f calculated from Eq. (5) are given in Fig. 3, together with those obtained from the well-known relation

$$c_f = 0.0363\,(k_s/\delta)^{0.251}\ [8],\quad c_f = 0.0139\,(x/k_s)^{-\frac{1}{7}}\ [9].$$

The explanation for the satisfactory agreement of the results given by Eq. (5) with those obtained from the relation of [8] and the disagreement with those given by the method of [9] may be found in that in [8] the dependence of the local friction coefficient c_f on the parameter k_s/x was obtained for a rough plane wall, whereas in [9] it was obtained by a power-law approximation of the logarithmic friction relation in experiments with rough tubes.

Using Eq. (3), Eq. (5) may be transformed to a "drag law for a rough plate" (in conditions of the complete appearance of roughness)

$$c_{f_0} = 0.0101 \left(\frac{\delta^{**}}{k_s}\right)^{-0.295}. \tag{6}$$

The above drag law for friction at an impermeable plate with pronounced surface roughness may be used within the framework of the well-known Kutateladze–Leont'ev concepts for the calculation of parameters of the gradient boundary layer at a rough permeable surface. The basic idea of the calculation method of [1] reduces to the use of a drag law written for "standard" conditions (for an isothermal flow of incompressible liquid at a smooth impermeable plate): to calculate the boundary-layer parameters in the given conditions, a correction is introduced to take account of the effect of "perturbing" factors (compressibility, nonisothermal conditions, permeability, pressure gradient), the numerical value of this correction being obtained for asymptotic conditions as $Re_0 \to \infty$.

Below, the "standard" conditions used are the same except that roughness of the plate is added, i.e., the initial drag law adopted is that for a rough plate in the form in Eq. (6).

In [1], the integral momentum relation for a plane turbulent boundary layer at a permeable surface in the form

$$\frac{d\mathrm{Re}^{**}}{dx} + (1 + H)\frac{\mathrm{Re}^{**}}{U_0}\frac{dU_0}{dx} = \mathrm{Re}_L \frac{c_{f_0}}{2}(\Psi + b) \tag{7}$$

was solved using the drag relation

$$c_{f_0} = B\,(\mathrm{Re}^{**}_{\mathrm{st}})^{-m} = B\,(\mathrm{Re}^{**})^{-m}\frac{\nu_0^*}{\nu_{\mathrm{st}}}. \tag{8}$$

When the drag law in Eq. (8) is replaced by a drag law of the form in Eq. (6), Eq. (7) takes the form

$$\frac{d\mathrm{Re}^{**}}{dx} + (1 + H)\frac{\mathrm{Re}^{**}}{U_0}\frac{dU_0}{dx} = 0.005\,\mathrm{Re}^{**^{-0.295}}\,\mathrm{Re}^{1.295}\left(\frac{k_s}{L}\right)^{0.295}(\Psi + b). \tag{9}$$

At constant surface temperature, Eq. (9) gives

$$\mathrm{Re}^{**}_{\mathrm{st}} = \exp\left[-\int(1 + H)\frac{d(\ln\bar{U}_0)}{d\bar{x}}\,d\bar{x}\right]\Bigg\{0.00324\,\mathrm{Re}_{0\,\mathrm{st}}^{1.295}\left(\frac{k_s}{L}\right)^{0.295} \times \int_{\bar{x}_0}^{\bar{x}}(\Psi + b)\,\bar{U}_0^{1.295}\exp\left[1.295\int(1 + H)\,\frac{d(\ln\bar{U}_0)}{d\bar{x}}\,d\bar{x} + C\right]\Bigg\}^{0.772}. \tag{10}$$

An obvious test of the soundness of the conception underlying the method adopted, in the context of calculations of the boundary layer at a rough permeable surface, is to establish whether it confirms the experimentally observed decrease [10] in the effect of surface roughness with increase in blowing rate. If the right-hand side of Eq. (9) is transformed – bearing in mind the relation $\Psi = [1 - (b/b_{\mathrm{cr}})]^2$ [1] – to the form

$$0.005\,\mathrm{Re}_L\left[\mathrm{Re}_L^{0.295}\mathrm{Re}^{**}\left(\frac{k_s}{L}\right)^{0.295}\left(1 - \frac{b}{b_{\mathrm{cr}}}\right)^2 + \bar{j}\,\right],$$

the degeneration of the influence of roughness as $b \to b_{\mathrm{cr}}$ becomes obvious.

For a given velocity distribution $U_0 = U_0(x)$ at the external edge of the boundary layer forming at the surface in the flow, and a given distribution of the injected-gas flow rate over the investigated surface, the calculation of the boundary-layer characteristics is made in two stages. In the first stage, the Reynolds number Re^{**} is calculated from a relation obtained [1] for the case of flow around an impermeable smooth surface, and then c_{f_0} is calculated from Eq. (8), this being necessary for the determination of the permeability parameter $b = j(2/c_{f_0})$. In the second stage, Re^{**} is found from Eq. (10), and this result is used to obtain the form parameter

$$f = \frac{\mathrm{Re}^{**}_{\mathrm{st}}}{\mathrm{Re}^{**}_{0\,\mathrm{st}}}\frac{dU_0}{dx}\frac{1}{U_0}$$

necessary to determine the position of the point of flow breakaway from the surface.

To verify the efficiency of the method, the contour-loss coefficient of a turbine-contour lattice was calculated from the equation $\varsigma_{\mathrm{co}} = \varsigma_{\mathrm{fr}} + \varsigma_{\mathrm{ed}}$. Here ς_{fr} is the frictional-loss coefficient, $\varsigma_{\mathrm{fr}} = 2\,(\delta^{**}_{\mathrm{ba}} + \delta^{**}_{\mathrm{conc}})/t\sin\beta_2$ [11]; ς_{ed} is the edge-loss coefficient, $\varsigma_{\mathrm{ed}} = k\Delta_{\mathrm{eff}}/a_2$ [12].

The values of $\delta^{**}_{\mathrm{ba}}$ and $\delta^{**}_{\mathrm{conc}}$ were calculated from Eqs. (6) and (10) using an M222 computer and a program written in Fortran.

The calculated and experimental values of the contour-loss coefficient are compared in Fig. 4.

NOTATION

R_z is the degree of roughness, μm;

$H_{i\,max}$, $H_{i\,min}$	are the absolute height of the five largest protuberances and absolute depth of the five largest depressions of the profile within the limits of the basic measurement length for the roughness, μm;
k_s	is the "equivalent" (sand) roughness, μm;
$\overline{U}_{0_1} = U_0/U_{0_1}$	is the dimensionless velocity, where U_{0_1} is the velocity at the outer edge of the boundary layer in the initial cross section $x = x_0$;
$\overline{x} = x/L$	is the dimensionless longitudinal coordinate, where x is the coordinate directed downstream along the surface in the flow, m, and L is the length of the surface in the flow, m;
y	is the coordinate directed along the normal to the surface, m;
u	is the longitudinal component of the mean velocity, m/sec;
δ	is the thickness of the dynamic boundary layer, m;
δ*	is the displacement thickness, m;
δ**	is the momentum-loss thickness, m;
$H = \delta^*/\delta^{**}$	is the velocity-profile form parameter;
n	is the coefficient in power-law approximation of velocity profile in boundary layer;
ν_0, ν_0^*	are the kinematic viscosity at the outer edge of the boundary layer determined from T_0 and T_0^*, respectively, m^2/sec;
ν_{st}^*	is the kinematic viscosity determined from T_{st}^*;
$Re - U_0x/\nu_0$; $Re_L = U_0L/\nu_0^*$, $Re^{**} = U_0\delta^{**}/\nu_0^*$, $Re_{0st} = U_0L/\nu_{st}$ and $Re_{st}^{**} = U_{0_1}\,\delta^{**}/\nu_{st}$	are the Reynolds numbers;
B, m	are the constants in the power-law approximation of the friction law in standard conditions;
$\overline{j} = \rho_{st}W_{st}/\rho_0U_0$	is the relative mass velocity through the body surface, where ρ_{st}, ρ_0 and W_{st}, U_0 are the density and velocity at the wall and at the outer edge of the boundary layer;
$\Psi = (c_f/c_{f_0})Re^{**}$	is the relative friction law, where c_{f_0} is the friction coefficient for "standard" (isothermal flow of an incompressible liquid at a smooth impermeable plate) conditions and c_f is the friction coefficient in real (taking account of all perturbing factors) conditions for the same Re**;
b_{cr}	is the critical value of the permeability parameter;
δ_{ba}^{**}, δ_{conc}^{**}	are the momentum-loss thickness at the back edge of the blade and on its concave side (at breakaway points on the exit edge), m;
$\overline{g_0} = g_0/G$, $g_0 = \oint \rho_{st}W_{st}dx$	is the coolant flow rate through the turbine-contour surface, kg/sec;
G	is the gas flow rate in the interblade channel;
x_{ro}	is the coordinate measured from the origin of plate roughness;
t	is the lattice step;
β_2	is the flow exit angle;
K	is the coefficient depending on the angle of rotation of the flow in the lattice;
Δ_{eff}	is the effective thickness of exit edge, determined by the position of the boundary-layer breakaway point;
a_2	is the orifice of interblade channel at outlet.

LITERATURE CITED

1. S. S. Kutateladze and A. I. Leont'ev, Heat and Mass Transfer and Friction in a Turbulent Boundary Layer [in Russian], Énergiya, Moscow (1972).
2. F. Bayley and G. Wood, "Aerodynamic performance of porous gas turbine blades," Aeronaut. J. R. Aeronaut. Soc., 73, No. 705 (1969).
3. N. D. Gryaznov, V. M. Epifanov, and V. I. Gus'kov, Results of Experimental Investigations of the Aerodynamic Characteristics of the Blade Units of Gas Turbine with Porous Cooling [in Russian], NIIINFORMTYaZhMASh Reference Collection, Energy Systems, 3-76-16 (1976).
4. H. Prechter, A. Schönbeck, and N. Scholz, Effusion Cooling of Turbine Blades, AGARD-CP-73-71 (1976).
5. N. K. Nikitin, Generalization of Semiempirical Turbulence Theory to Flow at Rough Surfaces with Different Conditions of the Appearance of Roughness Properties. Turbulent Flows [in Russian], Nauka, Moscow (1970).

6. V. M. Kovalenko, "Some characteristics of the wall region of a boundary layer at a rough plate," Izv. Sib. Otd. Akad. Nauk SSSR, No. 13, Issue 3 (1973).
7. A. I. Yakushev, Interchangeability, Standardization, and Technical Measurements [in Russian], Mashinostroenie, Moscow (1974).
8. K. K. Fedyaevskii and N. N. Fomina, "Investigation of the effect of roughness on the drag and the boundary-layer state," Tr. TsAGI, No. 441 (1939).
9. V. F. Droblenkov, "Turbulent boundary layer at a rough curvilinear surface," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 8 (1955).
10. V. P. Mugalev, "Heat transfer and flow at a rough porous surface with blowing," in: Heat and Mass Transfer [in Russian], Vol. 1, Part 2 (1972).
11. K. Bammert and H. Sonnenschein, "Der Einfluss verdickter und verdunnter Turbinenschaufeln auf die Gittereigenschaften," Arch. Eisenhuttenwes., 38, No. 4 (1967).
12. V. M. Epifanov, "Aerodynamic investigation of turbine lattices with thickened outer edges," in: High-Temperature Cooled Gas Turbines [in Russian], Mashinostroenie, Moscow (1971).

EXPERIMENTAL STUDY OF HEAT EXCHANGE IN HELIUM FLOW WITHIN A METAL – CERAMIC TUBE

L. L. Vasil'ev, G. I. Bobrova, and L. A. Stasevich

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Results are presented of an experimental study of the heat-transfer process in helium flow within a sintered porous bronze tube. The effect of flow suction rate through the porous wall upon temperature distribution and Nusselt number is demonstrated.

The problem of developing effective cooling methods is of great practical importance at the present time. A large number of studies has been presented on intensification of heat exchange, in particular, an intensification by suction of a portion of a flow through a porous channel wall.

A review of the available literature shows that heat exchange for liquid flow in permeable channels of various geometries has been studied to determine whether the heat-exchange process can be intensified by suctioning a portion of the flow through the permeable wall. The major portion of the works published analyze this process theoretically, usually assuming a laminar flow regime. This is apparently because theoretical analysis of a laminar flow is simpler and more concrete than treatment of a turbulent regime. Only a few experimental studies are available.

Analysis of heat exchange for a steady-state laminar flow proceeds from the system of equations of motion, continuity, and energy. Approximately 30 years ago Sellars [1] and Berman [2] demonstrated that the equations of motions for a laminar, completely developed flow with extraction or injection of a portion of the flow through a permeable porous wall may be reduced to a fourth-order nonlinear differential equation.

These studies were then expanded and refined in [3, 4].

Since suction through the wall intensifies heat exchange in flows in porous channels, investigators have concentrated their efforts not only on solving the hydrodynamic problem, but also on determining the effect of suction on temperature distribution over the radius and length of the porous tube.

Thus, in [3] the authors considered the effect of low draft velocities through a permeable tube wall into the main liquid flow within the tube upon the tube wall temperature distribution. In [5] temperature profiles were calculated for a flow moving within a porous tube, with the wall temperature of the tube maintained constant. Raithby [6] studied hydrodynamics and heat exchange for constant wall temperature and thermal flux, and analyzed the effect of individual parameters such as draft, suction, channel geometry, etc. on the temperature and velocity profiles.

A. V. Lykov Institute of Heat and Mass Exchange, Academy of Sciences of the Belorussian SSR, Minsk.
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